## Abstract

Active plasma resonance spectroscopy (APRS) is a process-compatible plasma diagnostic method that utilizes the natural ability of plasmas to resonate near the electron plasma frequency. The *Multipole Resonance Probe* (MRP) is a particular realization of APRS that has a high degree of geometric and electric symmetry. This radio-frequency driven probe of the spherical design is used for the supervision and control of low-temperature plasmas. The principle of the MRP can be described on the basis of an idealized geometry that is specifically suited for theoretical investigations.

Over the last decade, many studies of the MRP have been conducted to understand the resonance behavior of the plasma via the cold plasma model. However, in a pressure regime of a few Pa or lower, kinetic effects become important, which cannot be predicted by the cold plasma model. Therefore, in this work, a dynamic model of the interaction of the idealized MRP with a plasma is established, which is named the spectral kinetic model. Specifically, the self-consistent system is described in the Hamiltonian formalism, and the Poisson problem is solved explicitly with a Green's function. The proposed scheme reveals the kinetic behavior of the plasma that is able to emphasize the influence of kinetic effects on the resonance structure. Similar to *particle-in-cell*, the spectral kinetic method iteratively determines the electric field at each particle position, however, without employing any numerical grids. The optimized analytical model ensures the high efficiency of the simulation. Basically, the presented work is expected to cover the limitation of the cold plasma model, especially for the determination of the pure collisionless damping caused by kinetic effects. Notably, with the help of the spectral kinetic scheme, those energy losses can be explicitly predicted. It enables obtaining the electron temperature  $T_{\rm e}$  from the half-width  $\Delta \omega$  in the simulated resonance curve. That is, a formula to determine the electron temperature from the half-width is presented, which was discussed for years but unclarified. Besides, the electron density  $n_{\rm e}$  can be simultaneously derived from the resonance frequency.

Furthermore, the Monte Carlo collision model is integrated into the spectral kinetic model for a realistic simulation. The simulation results are compared with the measurement in a cylindrical double inductively coupled plasma reactor, where the corresponding plasma parameters are validated by reference measurements with a Langmuir probe. Eventually, good agreements in the comparison between the kinetic simulation and the experiment demonstrate the suitability of the presented scheme. The limitations of the cold plasma model are covered by the spectral kinetic model, and the pronounced kinetic effects in the low-pressure plasmas are well explained. Consequently, the spectral kinetic model can be seen as indispensable support in the MRP-plasma system for reliable supervision and control of the plasma process.

# **1** Introduction

#### 1.1 Plasma Diagnostics

Plasma science has gained increasing attention over the last decades, which is known as a multidisciplinary research area. Specifically, it is often a critical component of many disciplines, including astrophysics and space science, spectroscopy, surface and interface physics, and biomedical physics. Without doubt, plasma physics embraces almost the full breadth of the subfields in physics, and it also leads to the future innovation and development of these fields. As one of the widest utilized applied sciences, the diversity of its industrial application is remarkable, such as the well-known semiconductor manufacturing, intricate surface processing, plasma sterilization and disinfection, and realization of controlled nuclear fusion, just to name a few. In other words, plasma science has an extraordinarily broad impact in numerous industrial fields. The study of plasma technology is therefore of great interest within the general field of plasma science [1,2].

The term plasma was first introduced by Irving Langmuir in the context of ionized gases in 1929 [3]. A plasma is a partially or completely ionized gas exhibiting collective behavior, which contains electrons, ions, and neutrals. In general, the plasma density n and the electron temperature  $T_{\rm e}$  are essential for the characterization of a plasma. The plasma density usually refers to the number of charged species per unit of volume in the plasma bulk, and the electron temperature directly depends on the electron energy. Both parameters can vary significantly [4, 5]. Central to both experiment and theory is the accurate measurement of those key plasma parameters. Diagnostics are not only advantageous for the understanding of plasmas in the process, which provide better models and related simulations, but also beneficial for the process control in industrial applications. Therefore, one of the major topics to be investigated in plasma technology is the supervision and control of plasma. Figure 1.1 shows a schematic block diagram of the process control system. In order to achieve predictable and reproducible process results, many diagnostic techniques are proposed, especially due to the growing demand in industrial applications. However, only a few are applicable in technical plasmas and able to fulfill the industrial requirements. Thus, process control employing diagnostic systems still remains a challenging and vivid discipline. The diagnostic system must be robust and stable. Besides, it must minimally perturb the plasma process and be insensitive against perturbation by the process. Moreover, particularly important for industrial applications is the economical price of diagnostic systems. Furthermore, the spatially and time-resolved measurements of the controlled plasma process with a fast evaluation are required.



Figure 1.1: Control loop with disturbance of the process: The controller monitors the controlled process via a sensor. Depending on the captured system status, the adjustment can be executed by the actuator. The indicator sets the goal of the process, whereas the recorder collects the data.

The effort has been made by empirical optimization of plasma process to find the defined external parameters, such as voltage or current, which can be manipulated over the whole process according to the requirements. Unfortunately, controlled external parameters can cause a drift in the plasma process [6]. Thus, the internal parameters, such as the electron density or the electron temperature, need to be taken into account to closely monitor and adjust the plasma process.

### 1.2 Active Plasma Resonance Spectroscopy

Many different existing techniques are available for measuring the spatial profile and determination of various plasma parameters. Only few are suitable for industrial settings. One of the well-known concepts to industry-compatible plasma diagnostics is the so-called active plasma resonance spectroscopy (APRS). The method can be used in areas from moderate to very low pressures, which is often given in the industrial plasmas. The precise control is achieved by in-situ diagnostics. The concept of APRS is to utilize the natural ability of plasmas to resonate on or near the electron plasma frequency  $\omega_{\rm pe}$ , which was initially investigated back in 1929 [7]. In Figure 1.2, the idea of APRS is depicted: a signal is coupled into the plasma via a radio-frequency (RF) fed probe, and the response of the plasma is recorded in a certain frequency range [8]. Then some important plasma parameters can be determined by a mathematical model.



*Figure 1.2:* Schematical depiction of APRS: a radio-frequency signal is coupled into plasma via a probe, and the spectral response is recorded and analyzed. From the resonance curves, the corresponding plasma parameters can be determined by using a mathematical model.

To be more specific, the electron plasma frequency can be expressed in terms of the electron density,

$$\omega_{\rm pe} = \sqrt{\frac{e^2 n_{\rm e}}{\varepsilon_0 m_{\rm e}}}.$$
(1.1)

According to this definition, since the elementary charge e, the vacuum permittivity  $\varepsilon_0$ , and the electron mass  $m_e$  are constants, the electron density can be calculated if the electron plasma frequency is identified [9, 10]. Similarly, the ion plasma frequency  $\omega_{\rm pi}$ relates to the ion mass  $m_i$  and the ion density  $n_i$ ,

$$\omega_{\rm pi} = \sqrt{\frac{e^2 n_{\rm i}}{\varepsilon_0 m_{\rm i}}}.\tag{1.2}$$

However, the ions are much heavier than the electrons  $(m_i \gg m_e)$ , which causes a much smaller ion plasma frequency  $(\omega_{pi} \ll \omega_{pi})$ . It indicates that the electron plasma frequency is of particular importance to plasma dynamics. The proportionality of the electron plasma frequency and the resonance frequency  $\omega_{res}$  can be assumed, which writes

$$\omega_{\rm res}^2 \propto \omega_{\rm pe}^2 \propto n_{\rm e}.$$
 (1.3)

Therefore, from the obtained resonance response, the resonance frequency of the plasma can be evaluated in a mathematical model. Additionally, the electron temperature  $T_{\rm e}$  is linked to the collision frequency  $\nu$ . It can also be interpreted from the measurements. Hence, the determination of these important internal plasma parameters can be directly achieved in this method.

#### 1.3 Multipole Resonance Probe

Plasma probes are, in most cases, easy to use and give access to plasma parameters. The most renowned diagnostic probe is the Langmuir probe (LP), which was developed in the 1920s [11]. A Langmuir probe consists of a bare wire or an isolated metallic. To ensure reliable results, the tip must be conditioned so as not to interfere with the plasma nor be destroyed by it. The current can be measured at various applied voltages to obtain the current-voltage (I-V) characteristics [12]. This time-honored method is used in a wide variety of industrial or laboratory plasma devices. The theory of LP is well documented [13, 14]. In principle, plasma characteristics, such as  $n_{\rm e}$ ,  $T_{\rm e}$ , are extracted from the I-V curves by the LP system. However, it often presents several challenges regarding the complexity of the processes [15, 16].



*Figure 1.3:* Prototype of the MRP: The probe consists of two metallic hemispheres. The total diameter is 8mm. It is symmetrically driven via a tapered balun transformer, and it can be covered with a dielectric in a cylindrical quartz tube.

For a reliable real-time measurement of the internal plasma parameters, the multipole resonance probe (MRP) was proposed in 2008 as one of the realizations of APRS [17]. The device is a radio-frequency driven probe of a particular spherical design, which is powered in a frequency range approximately between 100 MHz and 10 GHz. The prototype of the MRP is shown in Figure 1.3 [18]. It consists of a spherical probe head and a holder. The MRP's head is comprised of two conducting metallic hemispheres, which constitute the electrically symmetric electrodes. These two hemispheres are separated by a dielectric layer, and fixed to a holder that contains the RF-supply. The probe is inserted into a quartz tube in order to isolate it from plasma. The setup of the MRP provides two important features: its electrical behavior is symmetric regarding the mapping, and also its geometry is approximately symmetric.