1 Introduction

1.1 Structural Vibrations

Vibrations embrace a broad range of topics that address problems from particle physics to multibody motion in different systems. Mechanical systems in particular are subject to constant development due to their increasingly complex structures and ever-increasing system requirements. Variable system properties that change over time, such as variable structures with moving mass-bearing parts and deformations, are examples of such developments. Recent developments in materials science and the fabrication of soft sensors and actuators are opening up entirely new possibilities for biologically inspired designs for mechanical assemblies. In structural mechanics, the focus is on lightweight structures designed for ideal, load-oriented use of materials, as often found in nature. Lightweight construction can also meet the conflicting requirements of high stability and safety of structures despite the lightweight components. It is also an essential technology for addressing the increasing challenges of energy and resource efficiency. Weight reduction has a direct impact on energy efficiency, especially in applications involving moving objects such as aircraft, vehicles and manipulators, as well as in the transportation of goods. The main focus is on improving motion while minimizing energy without compromising performance. The lightweight design has a significant impact on direct material costs and reduces fuel consumption in the aerospace and automotive industries. In addition, production lines and goods transport benefit from these developments to reduce production, transport and disposal costs. On the other hand, it is clear that mass reduction technology and slender structures increase mechanical vibrations. For complex structures, the mechanical vibrations become a bottleneck factor and spawn new branches of challenges. In the wide range of solid mechanics, the system classification used in [Tri+08] distinguishes between systems based on materials and the number of Degrees of Freedom (DoF) in four classes: Non-redundant, Redundant, Hard Continuum, and Soft. As Figure 1.1 shows, this work is concerned with the hard continuum class, specifically the class of systems that are both flexible and rigid, such as a flexible system with variable mass structure.

Parametric Resonance

Oscillatory behaviors can be classified according to their formation mechanism into; free, self-excited, coupling, forced and parametric oscillations [MPS16]. Free oscillations, also called self-excited oscillations, are motions of an oscillatory system that are not subject to

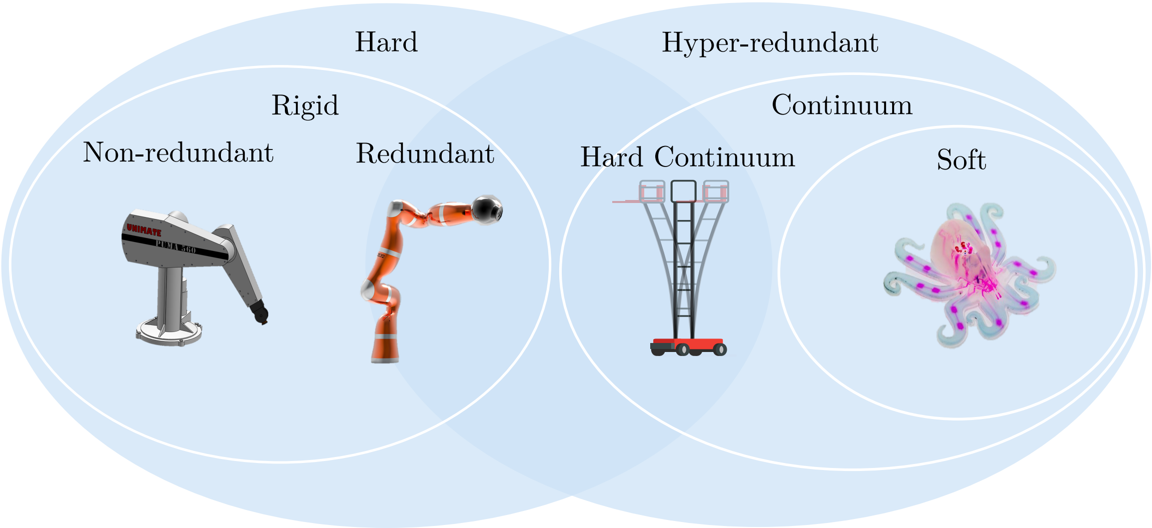

Figure 1.1: Classification of solid mechanics based on materials and DOF

any external energy input. In contrast, self-excited oscillations are supplied with energy. The clock is a typical example with an external energy source. Coupling oscillations result when two oscillatory systems mutually influence each other or when one oscillatory system has multiple degrees of freedom. An example of coupling oscillations are multi-mass spring-damper systems interacting with each other. Forced oscillations are subject to external excitation. The differential equations of forced oscillations contain a time-dependent excitation term. An example of forced oscillation is given by systems excited by a periodic input, i.e. mass-damper system excited by rotating unbalances. Parametric oscillations result from time-varying, mostly periodic parameters in a system [FN11]. Parametric excitation can be external or internal and can cause a change in one or more parameters. From a mathematical perspective, parametric oscillations contain time-dependent, mostly periodic coefficients in the differential equations. One of the characteristics of parametric oscillations is the fact that the excitation has no effect on the oscillatory system as long as it remains in the equilibrium position. However, this equilibrium position can become unstable under certain conditions, especially at certain ratios of the natural frequency to the excitation frequency, so that any disturbance, however small, can trigger the buildup of parametrically excited oscillations [MPS16]. A typical example is a swing, in which the body pose plays an essential role in the propagation of the amplitude. To swing, one shall pull together in the middle position and straighten up on the extreme positions. With this, one performs oscillations with a frequency, which has an impact on the swing oscillation. The steadily shift of the body's center of gravity changes the distance between the pivoted and the carrying mass. This can be seen as a pendulum with varying rope length [Sey04]. One of the main characteristics of an oscillatory system is the resonance phenomena that occur in all types of oscillations. Resonance occurs when the excitation frequency is equal or close to the natural frequency of a system and results in increased amplitude. Depending on the nature of a system, resonant frequencies can occur in very complicated forms.

Impact of Nonlinearities

The linear superposition principle used for linear differential equations no longer applies to nonlinear systems. As a result, nonlinear dynamical systems exhibit much more complex behavior. Parametric excitation can lead to parametric instability, even without additional external excitation. [DES99] showed that instability is a general property that can occur in many systems with coupled oscillators or with nonlinear coupling. In instability, a small perturbation of the normal mode causes a finite amount of energy to be transferred to the other mode. The instability of the mass-spring system is explained as an internal parametric excitation, where the longitudinal motion drives the transverse motion parametrically. Nonlinear oscillators exhibit frequencies that are generally amplitude-dependent and consist of frequencies that are integer multiples or fractions of the fundamental frequency. For nonlinear systems, the resonance can be characterized as parametric resonance, superharmonic resonance, or multi-harmonic [Che14; LR12; Fuc14; HC11]. Such a resonance characterization shows when a system can achieve bistability, hysteresis and even instability. Parametric resonance is an exciting phenomenon that can be used to study the effects of parameter changes on the stability of a system. Unstable systems can be stabilized by specific excitations. One of the best known examples is the Mathieu equation, a linear differential equation with a time-dependent coefficient. Recently, some mathematical methods have been developed to solve the nonlinear differential equations of oscillators. Most of them are applicable only to a certain type of nonlinearity and do not apply in general [Cve18]. Even the numerical analysis of the linear Mathieu equation is possible only because of the coefficient, which is periodic in time. Parametric resonance makes the analysis and control of general nonlinear systems notoriously difficult.

1.2 Application Examples

Parametric resonance occurs in a broad class of systems that typically exhibit either time-varying or nonlinear dynamics. In contrast to electrical or fluid systems, time-varying quantities are more common in mechanical systems.

Power Grid

As more and more renewable energy producers participate in the power grid, the grid impedance between producer and consumer nodes changes due to changes in availability. This leads to a variable resonance frequency in the entire grid, which makes the design of passive filters in the grid more difficult. In practice, in such cases, the filters are designed with a higher and wider damping factor to counteract this change. However, this attenuates a broad spectrum, which has a great impact on the efficiency.

Oscillatory Electrical Circuit

Parametric amplifiers use excitation to enhance the response of a resonance-controlled oscillator in different frequency ranges. Parametric amplifiers are typically used in acoustics and optical spectral applications. Such amplifiers are frequency selective and consist of RLC circuits with time-varying elements, such as an inductor using a dynamo. Non-degenerate parametric amplifiers exhibit a nonlinear charge-voltage characteristic due to voltage-dependent capacitance.

Ship Motions

Ships are exposed to external parametric excitations from waves. Rolling resonance is a thorny problem in shipping with very costly and damaging consequences [FN11]. When longitudinal wave excitation frequencies are approximately twice the rolling frequency, ships experience large rolling oscillations that can cause instability in the form of ship overturning. Depending on the ship length and speed, the parametric resonance varies and parametric rolling can be avoided.

Autonomous Aerial Refueling

Another example is the docking process during autonomous air refueling with a flexible hose. Depending on the tanker's speed, drag vortex, turbulence, air flow and closure speed, the flexible refueling hose experiences transverse, longitudinal and torsional vibrations. Such docking is extremely difficult due to hose vibrations, the frequency of which depends on various factors such as hose length. [MB17] analyzed the frequency behavior and many other studies have been devoted to vibration control, such as [SXL19], to avoid serious refueling accidents.

Stacker Crane

In order to study the parametric resonance phenomenon in detail with a concrete example, this thesis uses a flexible stacker crane with a nonlinear system equation. Stacker Cranes (STCs) are highly dynamic, rail-guided, planar, flexible robotic manipulators. STCs are characterized by a long mast and a load handling device, as shown in Figure 1.2. With the rapid growth of fully automated production facilities, STCs have become a common attribute as part of automated guided vehicles in fully automated warehouses and logistics centers. They consist of a drive unit, a lifting unit and a loading unit and move in three axes. The drive unit moves longitudinally in narrow aisles, the lift unit moves vertically and the load unit moves transversely in the aisles. STCs perform fast and accurate positioning maneuvers for loading, lifting, transporting and unloading. Some STCs (e.g. Viastore) are

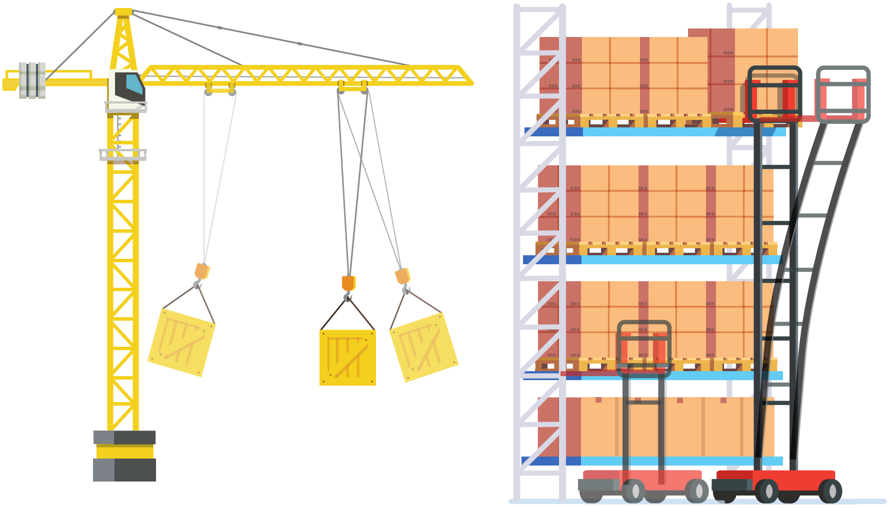

Figure 1.2: Parametric resonance is a common phenomena in mechanical systems with time-varying structures as in gantry cranes with variable cable length or stacker cranes with variable load change

designed to reach a height of 45 m and transport loads up to 3000 kg. Mini-load STCs are designed to achieve extremely high speeds of up to 6.5 m/s for movement in the horizontal direction and up to 3.5 m/s) for movement in the vertical direction, as given in [LER14]. Primarily promoted from the mast flexible and slender form, undesired vibrations arise, especially under high speeds. These vibrations increase material fatigue and reduce productivity, efficiency and positioning accuracy. In addition, long-term material wear leads to enormous reinvestment and maintenance costs. To increase the efficiency of such a system, vibration damping must be combined with fast dynamic positioning. A load moving along a flexible structure poses an additional problem, as this causes the phenomenon of parametric resonance, making vibration reduction dependent on the current load characteristics. Similar problems exist in many flexible systems with variable structures, such as gantry or boom cranes with variable cable lengths [KR10], as shown in Figure 1.2.

1.3 Main Objective

Since parametric resonance is a vast topic with a multitude of challenges, this thesis approaches the subject using a specific example of STC and the parametric resonance phenomenon therein. For such a system, determining a control approach to reduce the time-varying oscillations is challenging and requires accurate frequency analysis and a precise mathematical model. A suitable mathematical model should have the feature that the oscillatory behavior is accurately preserved when modeling the parametric resonance, while the control approach should use the knowledge of the resonance to actively incorporate it in the design of the control damping. Parametric resonance analysis and control of an