Contents

1. Introduction **1**

2. Onset of synchronization in networks of noisy phase oscillators **5**
2.1. Coarse-graining uncorrelated random networks 6
2.2. The nonlinear Fokker-Planck equation 8
2.3. The critical coupling strength 11
2.4. Application to dense small-world networks 14
2.5. Interpolating between sparse and dense networks 22
2.6. Networks with degree-dependent frequency dispersion 24
2.7. Implementation of an example with first numerical experiments 25
2.8. Detailed analysis and clarification of noise effects 29
2.9. Summary and outlook 35

3. Approximate solution to the stochastic Kuramoto model **37**
3.1. Exposition of the Gaussian approximation 37
3.2. Time-dependent solutions and long-time limits 40
3.3. Gaussian theory vs. numerical experiments and exact results 42
3.4. Temporal fluctuations vs. quenched disorder 45
3.5. Including complex networks 46
3.6. Summary and outlook 47

4. Excitable elements controlled by noise and network structure **49**
4.1. Noise-driven active rotators 49
4.2. First example: regular networks 51
4.3. Second example: binary random networks 54
4.4. Cooperative behavior between self-oscillatory and excitable units 57
4.5. Bifurcation diagrams and order parameters for a concrete example 59
4.6. Summary and outlook 65

5. Noisy oscillators with asymmetric attractive-repulsive interactions **69**
5.1. Bifurcation diagrams 72
5.2. Simulation results 76
5.3. Towards an arbitrary number of populations 77
5.4. Summary and outlook 79

6. Synchronization in the stochastic Kuramoto-Sakaguchi model **81**
6.1. Application of Gaussian approximation 81

6.2. Connection between common frequency and synchronization 82
6.3. Towards a new phenomenological theory 83
6.4. Summary and outlook 85

7. Conclusions **89**

A. Directed networks **93**

B. Beyond uncorrelated networks **95**
B.1. Two-point correlated random networks 96
B.2. Assortativity by degree 98
B.3. Assortativity by frequency 98

Bibliography **101**